**Stark & Zeeman Interaction**

Now looking at entire material, i.e., dielectric/dimagnetic, immersed in constant EM field.

**H for Dielectric/Dimagnetic material (in an external EM Field)**

Well now I want to consider an entire material with some possible dielectric/dimagnetic response. Also going to want to put H in terms of the free/external fields since those are the thermodynamically appropriate variables (see Thermodynamics folder/Equilibrium Systems). So the Hamiltonian (see EM folder where we derived an action for particles and fields), we found that H was (summing over all atoms and all electrons within atoms, in our material):



Note that now, b refers to bound charges within the dielectric, and f refers to free charges without it, i.e., residing on the capacitor plates, or the solenoid, say. And **A** = **A**f + **A**b and **B** = **B**f + **B**b. And φf is the electric potential coming from the free charges. The ∫dτ Eb2 term will give us the internal electrostatic potential energy within the material. This will include the potential energy within the atom, and between atoms.



where the first term is the sum of all electric potential energies between the nucleus and electrons within an atom, the second term is the electric potential energy between all the electrons within an atom. The third term is the electric potential energy between atoms themselves. There are a few options for this guy. If the atoms have a net electric dipole moment, then maybe we would approximate this with the dipole-diple interaction expression:



where ρb is the bound charge density, **P**(r) is the polarization density, and the last term is what this works out to if the polarization density is uniform (see EM folder/Dipole-Dipole interaction). Recalling our result in previous file for an atom’s dipole moment, we say that the dipole moment density is given by:



where I is measured from the atom’s nucleus. Maybe if there isn’t an electric dipole moment, then some variation on the exchange interaction expression would be appropriate (see previous file)?



Then we have the ∫dτ Bb2 term. This will give us the internal magnetostatic potential energy so-to-speak within the atom itself, and between atoms. We ignored the intra-atomic contribution last time and will again, because it’s insignificant. But, since we’re interested in dimagnetic susceptibility effects whereby the atom may become magnetized, we will include the magnetostatic potential energy that exists between atoms by virtue of their magnetization. So,



where **j**b(r) is the bound charge. Might check out EM folder/Energy Thoughts for how that top line happens, and then EM folder/Dipole-Dipole for the second line. And the last line is what we get if we can presume the magnetization is uniform. From our work in previous file, note that we can posit the magnetization density as being,



where ⊥i is measured from the atom’s nucleus. **B** is the local field, just as **B**f was the local field in the previus file. ‘tis unfortunate that **M** itself would depend on **B**. So now we have:



To a good degree of approximation, we can write the last guy as, Taylor expanding the φf(r) potential:



and, before proceeding, I’m going to recognize this guy in brackets as the polarization operator. So filling this in, neglecting the constant (atoms aren’t moving) we now have:



I guess at this point we could attempt to expand the kinetic energy term, i.e., the |**p**-e**A**|2/2m – gγ**S**·**B** guy. We would presume to know **A**f(r) and **B**f(r) = ∇×**A**f(r). And then we could write **A**b using the formula from the EM folder/Insulators:



and then



So can solve for **M** in terms of **B**, in terms of **M** and **j**f, So we can self-consistently get **M** in terms of **j**f, and hence in terms of **B**f, and then everything in terms of **B**f. But that would be messy.

**Specialization to constant fields**

It would seem better to just presume constant **B**, and **E** too, while we’re at it.



and copying all that work we did in the first section, we should ultimately get (within the lowest energy space of the CFA):



Now we’ll use the fact that in a uniform substance of something like solenoidal geometry, where we can equate **B**b = μ0**M**, **B**f = μ0**H**, and we can say, **B** = **B**f + μ0**M**. So,



Now M obeys the self-consistent equation,



and solving for **M** in terms of **B**f, we have:



So,



How big is that menacing term?



So looks like we should usually be able to ignore it, in comparison to 1. So then we can still just say,



and then we have:



where **M** is understood to be the operator above, and **P** the operator further up.

**Specializing to paramagnets/ferromagnets**

For primarily paramagnetic/ferromagnetic materials, the r⊥ term is usually quite small, and negligible. So then,



And then we could say,



**H for Dielectric/Dimagnetic material + EM Field**

Now we’ll do the Hamiltonian for a substance with the free fields included. So the Hamiltonian with fields included (see EM folder where we derived an action for particles and fields), we found that H was (summing over all atoms and all electrons within atoms, including the free ones):



where **B** and **E** are the total fields generated by both the bound charges in all the atoms of our substances, and the free charges aforementioned. I will want to put this in terms of **E**f and **B**, since these are the thermodynamically proper variables (see Thermodynamics/Equilibrium Systems). So I guess I’ll focus on unpacking the E terms. I will treat the free charges ‘classically’, and I’ll treat the bound ones ‘quantumly’. Since number of bound charges vastly outnumbers the number of free charges, I think we can restrict the sum over atoms, i, to just the bound ones, especially so since the kinetic energy of the free charges is either basically zero if they’re stuck on the plate, or even negligible if they’re forming a current. Now apropos the **E** field potential energy stuff, recall that in the EM folder (insulating energy) we wrote:



which looks a bit contradictory, but the terms are different. **E** on the left is the total field everywhere (both within the dipole and without), while **E** on the right is the bulk-interstitial field, i.e., just the field outside the dipoles, and **E**b+/-i are the individual fields of the tiny dipoles, or in our case, just the field within the atoms. We can write the bulk/interstitial field as **E** = **E**f + **E**b, and so:



The middle guy can be worked back into the form we expect:



And then we can expand φf in a Taylor series, like we did above, and write this as:



And we’ll copy our definition (ε0/2)∫dτ Eb2 = Vinteratomic(E), from above. So we have:



And filling this into H we have:



We can’t really go too much further since expressing A in terms of B is problematic when the latter is position-dependent. One nice thing we can do though is identify the polarization operator (which we already know).



**Specializing to constant fields**

It’d be more productive at this point to further specialize to a constant **B** field. And I’ll presume constant **E** field too, for symmetry. In that case, we can introduce the symmetric gauge as usual, and end up with,



Then apropos the magnetic moment, we can see that (see Thermodynamics/Equilibrium Systems or EM/Insulator Energy):



which is the same as we had before. I’m not sure of the utility of this, but maybe it’s intructive to see how we could break down the ∫dτB2 term, just like we did the ∫dτE2 term. So consider:



where the B on the left is the total field everywhere, inside and outside the dipole region, while B on the right is just the outside interstitial field, and the last term comprises the fields within the atom. We will ignore that contribution again. Expanding the RHS ∫B2 term,



The middle guy can be worked into:



[a similar set of manipulations could’ve put the ∫dτ**E**b·**E**f expression directly in terms of **E**f and **P**, but we wanted an expression for **P** so did it differently] And just copy our definition of (1/2μ0)∫dτ Bb2 = Vinteratomic(B) from above. So now we have:



in good analogy with what we found vis a vis the analogous ∫E2 term.

**Paramagnets/Ferromagnets**

For primarily paramagnetic/ferromagnetic materials, the r⊥ term is usually quite small, and negligible. So then,



And then we could say,

